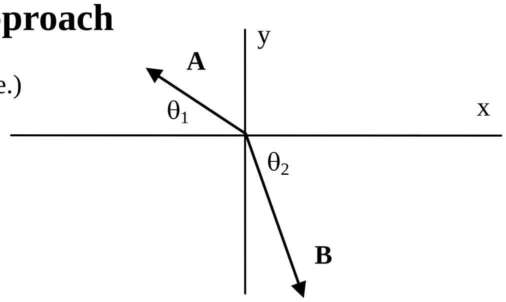
VECTOR ADDITION – General Approach

Problem 1 - Adding Two Vectors

Add the two vectors shown. (not drawn exactly to scale.)
Use
A = 5m
B = 10m
$\theta_1 = 30^o$
$\theta_2 = 80^o$



Angles

We can measure all angles from the +x axis.

Angles in the 1st and 2nd quadrants range from 0 to +180°.
Angles in the 3rd and 4th quadrants range from 0 to -180°.

In our examples we replace

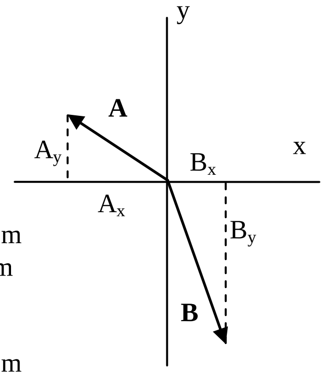
$\theta_1 = 30^o$ with $\boldsymbol{\theta_1 = +150^o}$
$\theta_2 = 80^o$ with $\boldsymbol{\theta_2 = -80^o}$

Analytical Solution

Write vector equation then **component equations**:

C = **A** + **B**

X: $C_x = A_x + B_x$
Y: $C_y = A_y + B_y$



Next fill in what each term is (the geometry is shown at right):

$A_x = A\cos\theta_1 = (5m)\cos(150^o) = (5m)(-0.866) = -4.33m$
$B_x = B\cos\theta_2 = (10m)\cos(-80^o) = (10m)(0.174) = 1.74m$

$A_y = A\sin\theta_1 = (5m)\sin(150^o) = (5m)(0.5) = 2.5m$
$B_y = B\sin\theta_2 = (10m)\sin(-80^o) = (10m)(-0.985) = -9.85m$

$C_x = A_x + B_x = -4.33m + 1.74m = -2.59m$
$C_y = A_y + B_y = +2.5m - 9.85m = -7.35m$

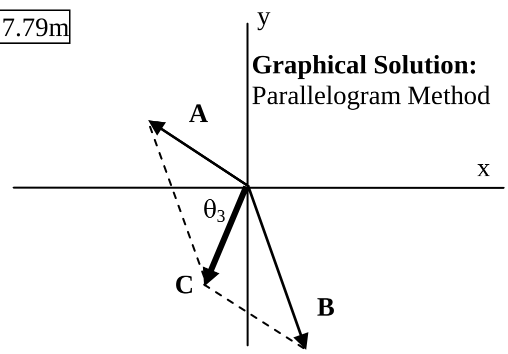
Magnitude: $C = \sqrt{C_x^2 + C_y^2} = \sqrt{(2.59m)^2 + (7.35m)^2} = \boxed{7.79m}$

Direction:

Draw the vector in the correct quadrant. In this problem the resultant vector, **C**, has negative x and y components, so it lies in the 3rd quadrant. We can also see this by drawing the resultant vector using the parallelogram method of addition of vectors, label its angle to the x-axis, θ_3, and calculate the angle from the components:

$\theta_3 = \tan^{-1}(C_y/C_x) = \boxed{70.6^o}$

Graphical Solution:
Parallelogram Method



Problem 2 - Adding Three Vectors

Add the three vectors shown. (drawing not exactly to scale.)

Use

A = 5m

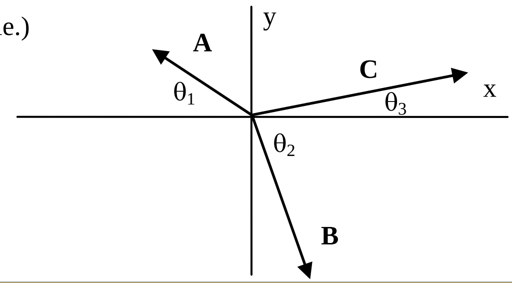
B = 10m

C = 15

$\theta_1 = 30^o$

$\theta_2 = 80^o$

$\theta_3 = 10^0$



Angles

We can measure all angles from the +x axis.

Angles in the 1st and 2nd quadrants range from 0 to +180^o.

Angles in the 3rd and 4th quadrants range from 0 to -180^o.

In our examples we replace

$\theta_1 = 30^o$ with $\boldsymbol{\theta_1 = +150^o}$

$\theta_2 = 80^o$ with $\boldsymbol{\theta_2 = -80^o}$

$\theta_3 = 10^o$ with $\boldsymbol{\theta_3 = +10^o}$

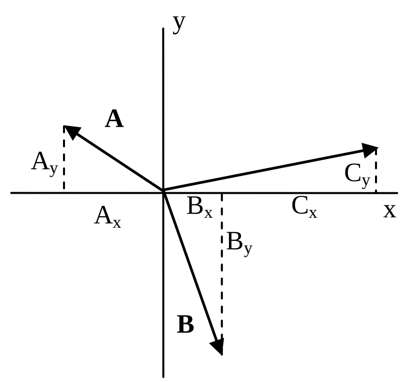
Analytical Solution

Write vector equation then component equations:

D= **A** + **B** + **C**

X: $D_x = A_x + B_x + C_x$

Y: $D_y = A_y + B_y + C_y$



Next fill in what each term is (the geometry is shown at right):

$A_x = A\cos\theta_1 = (5m)\cos(150^o) = (5m)(-0.866) = -4.33m$

$B_x = B\cos\theta_2 = (10m)\cos(-80^o) = (10m)(0.174) = 1.74m$

$C_x = C\cos\theta_3 = (15)\cos(10^o) = (15m)(0.98) = 14.8m$

$A_y = A\sin\theta_1 = (5m)\sin(150^o) = (5m)(0.5) = 2.5m$

$B_y = B\sin\theta_2 = (10m)\sin(-80^o) = (10m)(-0.985) = -9.85m$

$C_y = C\sin\theta_3 = (15)\sin(10^o) = (15m)(0.174) = 2.60m$

$C_x = A_x + B_x = -4.33m + 1.74m = -2.59m$

$C_y = A_y + B_y = +2.5m - 9.85m = -7.35m$

$D_x = A_x + B_x + C_x = -4.33m + 1.74m + 14.8m = 12.21m$

$D_y = A_y + B_y + C_y = +2.5m - 9.85m + 2.60m = -4.75m$

Magnitude: $D = \sqrt{D_x^2 + D_y^2} = \sqrt{(12.21m)^2 + (4.75m)^2} = \boxed{13.10m}$

Direction:
Draw the vector in the correct quadrant. In this problem the resultant vector, **D**, has a positive x component and a negative y component, so it lies in the 4th quadrant. We can also see this by drawing the resultant vector using the tail-to-tail method of addition of vectors, label its angle to the x-axis, θ_4, and calculate the angle from the components:

$\theta_4 = \tan^{-1}(D_y/D_x) =$ $\boxed{21.3^o}$