Sample Exam #2
Math311

Name____________________________________

ESSAY. Write your answer in the space provided or on a separate sheet of paper.

Find an explicit description of the null space of matrix A by listing vectors that span the null space.

1) $A = \begin{bmatrix} 1 & -2 & 3 & -3 & -1 \\ -2 & 5 & -5 & 4 & -4 \\ -1 & 3 & -2 & 1 & -5 \end{bmatrix}$

Determine if the vector u is in the column space of matrix A and whether it is in the null space of A.

2) $\mathbf{u} = \begin{bmatrix} -1 \\ -1 \\ 3 \end{bmatrix}$, $A = \begin{bmatrix} 1 & -3 & 4 \\ -1 & 0 & -5 \\ 3 & -3 & 6 \end{bmatrix}$

Determine whether $\{v_1, v_2, v_3\}$ is a basis for $\mathcal{R}^3$.

3) $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -3 \\ 5 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -3 \\ 8 \\ -2 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}$

Find a basis for the column space of the matrix.

4) Let $A = \begin{bmatrix} -1 & 3 & 7 & 2 & 0 \\ 1 & -2 & -7 & -1 & 3 \\ 2 & -4 & -9 & -5 & 1 \\ 3 & -6 & -11 & -9 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -3 & -7 & -2 & 0 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & 5 & -3 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

It can be shown that matrix A is row equivalent to matrix B. Find a basis for Col A.

Solve the problem.

5) Let $H = \left\{ \begin{bmatrix} a + 2b + 2d \\ c + d \\ -3a - 6b + 4c - 2d \\ -c - d \end{bmatrix} : a, b, c, d \text{ in } \mathcal{R} \right\}$

Find the dimension of the subspace H.

Find the dimensions of the null space and the column space of the given matrix.

6) $A = \begin{bmatrix} 1 & -3 & -5 & 3 & 0 \\ -2 & 1 & 3 & -4 & 1 \end{bmatrix}$

Assume that the matrix A is row equivalent to B. Find a basis for the row space of the matrix A.

7) $A = \begin{bmatrix} 1 & 3 & -4 & 0 & 1 \\ 2 & 4 & -5 & 3 & -1 \\ 1 & -5 & 0 & -3 & 2 \\ -3 & -1 & 8 & 3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & -4 & 0 & 1 \\ 0 & -2 & 3 & 3 & -3 \\ 0 & 0 & -8 & -15 & 13 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

For the given matrix A, find a basis for the corresponding eigenspace for the given eigenvalue.

8) $A = \begin{bmatrix} 1 & -5 & -5 \\ -5 & 1 & 5 \\ 5 & -5 & -9 \end{bmatrix}, \ \lambda = -4$

Find the eigenvalues of the given matrix.

9) $\begin{bmatrix} -7 & -4 \\ 33 & 16 \end{bmatrix}$

Diagonalize the matrix A, if possible. That is, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

10) $A = \begin{bmatrix} 5 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$