

Name:
High School:

Lock Haven University
Second Annual Mathematics Competition for High School Students
Wednesday November 11, 2009

Individual Test for Grades 11 and 12

Directions: *No calculators or reference materials are permitted. You may use a pencil, an eraser and your examination paper as scratch paper. Additional pencils and scratch paper will be provided if you raise your hand. In this examination all problems are multiple choice questions with five possible answers. Read each problem carefully and decide which is the best answer. You will receive one point for each correct answer.*

1. Find the next number in the sequence

1, 2, 11, 36, 85, \dots

A. 258 **B.** 166 **C.** 188 **D.** 170 **E.** none of A, B, C, or D

2. One solution to the equation

$$(2x + 5)^{\frac{2}{5}} = 2$$

is

A. $\frac{\sqrt[5]{2^2-5}}{2}$ **B.** $2\sqrt{2} - \frac{5}{2}$ **C.** $\frac{\sqrt[5]{4+5}}{2}$ **D.** $1 - 5^{\frac{5}{2}}$ **E.** neither A, B, C nor D

3. Find a possible value for b so that

$$f(x) = -2x^2 + bx - 13$$

has a minimum value of 5.

A. 10 **B.** 8 **C.** 12 **D.** 14 **E.** neither A, B, C, nor D

4. Given the function

$$f(x) = -3x + 2,$$

which of the following statements are true?

i: $f(4) = -12x + 2$

ii: $f(y + 1) = -(3y + 1)$

iii: $f(-4) = 14$

A. i: only **B.** ii: only **C.** iii: only **D.** i: and ii: only **E.** ii: and iii: only

5. If

$$\frac{17}{x} = \frac{11}{319},$$

then the digit in the tens place of x is:

A. 3 **B.** 9 **C.** 1 **D.** 5 **E.** neither A, B, C, nor D

6. Find the value of the following quantity:

$$1 - 2 + 3 - 4 + \cdots - 98 + 99$$

A. -50 **B.** -49 **C.** 0 **D.** 49 **E.** 50

7. Find the value of the following quantity:

$$2\left(1 - \frac{1}{2}\right) + 3\left(1 - \frac{1}{3}\right) + 4\left(1 - \frac{1}{4}\right) + \cdots + 10\left(1 - \frac{1}{10}\right)$$

A. 45 **B.** 49 **C.** 50 **D.** 54 **E.** 55

8. Each of the letters x, y, z and w represents a different letter in the set $\{1, 2, 3, 4\}$, but not necessarily in that order. If

$$\frac{w}{x} - \frac{y}{z} = 1,$$

then $w + y$ is:

A. 3 **B.** 4 **C.** 5 **D.** 6 **E.** 7

9. A $4 \times 4 \times 4$ cubical box contains sixty four identical small cubes that exactly fill the box. How many of these small cubes touch a side or the bottom of the box?

A. 48 **B.** 52 **C.** 60 **D.** 64 **E.** 80

10. If

$$\frac{x}{3-x} = \frac{y}{5-y} = \frac{z}{16-z} = 2$$

then

$$x + y + z =$$

A. 12 **B.** 15 **C.** 16 **D.** 18 **E.** $\frac{37}{2}$

11. If

$$1 - \frac{4}{x} + \frac{4}{x^2} = 0$$

then $\frac{2}{x}$ equals

A. -1 **B.** 1 **C.** 2 **D.** -1 or 2 **E.** -1 or -2

12. The number of distinct pairs (x, y) of real numbers satisfying both of the following equations:

$$x = x^2 + y^2, \quad y = 2xy$$

is

A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** 4

13. Find the area of the smallest region bounded by the graphs of $y = |x|$ and $x^2 + y^2 = 4$.

A. $\frac{\pi}{4}$ **B.** $\frac{3\pi}{4}$ **C.** π **D.** $\frac{3\pi}{2}$ **E.** 2π

14. If the ratio of $2x - y$ to $x + y$ is $\frac{2}{3}$, then the ratio of x to y is

A. $\frac{5}{4}$ **B.** $\frac{4}{5}$ **C.** 1 **D.** $\frac{6}{5}$ **E.** $\frac{1}{5}$

15. A positive number x satisfies the inequality

$$\sqrt{x} < 2x$$

if and only if

A. $x > \frac{1}{4}$ **B.** $x > 2$ **C.** $x > 4$ **D.** $x < \frac{1}{4}$ **E.** $x < 4$

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Tie Breaker Question: If

$$\frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\ddots}}}}}$$

were to be defined as a real number, then what should its value be?