

Name:

High School:

3rd Annual Lock Haven University
Mathematics Competition for High School Students
Thursday November 18, 2010 from 9:45 - 10:20 a.m. (35 minutes)

Individual Test for Grades 11 and 12

Directions: *No calculators or reference materials are permitted. You may use a pencil, an eraser and your examination paper as scratch paper. Additional pencils and scratch paper will be provided if you raise your hand. You will receive one point for each correct answer.*

1. A student wrote that the product of $a + i$ and $b - i$ was $a + b + i$, where $i^2 = -1$. If this was correct then the minimum value of ab is
 - (a) 0
 - (b) 1
 - (c) 2
 - (d) -2

2. The sum and product of two numbers x and y are each equal the same positive value. Then the difference $y^2 - x^2$ is
 - (a) 0
 - (b) $\frac{(x-1)^2}{x^2}$
 - (c) $\frac{x^2}{(x-1)^2}$
 - (d) $\frac{(2-x)x^3}{(x-1)^2}$

3. When the last digit of a certain 6 digit number N is transferred to the first position, the other digits moving one place to the right, the new number is exactly one-third of N . The sum of the six digits of N is
 - (a) 28
 - (b) 27
 - (c) 26
 - (d) 25

4. The value of $25^{\log_5 6}$ is
 - (a) 36
 - (b) 6
 - (c) 5
 - (d) $\log_5 6$

5. The distance between the two complex numbers $1 - 2i$ and $2i - 2$ is
- $3 - 4i$
 - $-3 + 4i$
 - 5
 - $\sqrt{17}$
6. If $F(n + 1) = \frac{2F(n)+1}{2}$ for $n = 1, 2, \dots$, and $F(1) = 2$, then $F(101)$ equals
- 49
 - 50
 - 51
 - 52
7. If $\frac{4^x}{2^{x+y}} = 8$ and $\frac{9^{x+y}}{3^{5y}} = 243$, with x, y real numbers, then xy equals
- 12
 - 6
 - 4
 - $\frac{12}{5}$
8. If b people take c days to lay f bricks, then the number of days it will take c people working at the same rate to lay b bricks, is
- fb^2
 - $\frac{b}{f^2}$
 - $\frac{f^2}{b}$
 - $\frac{b^2}{f}$
9. When $x^3 + k^2x^2 - 2kx - 6$ is divided by $x + 2$ the remainder is 10, then k must be
- 2 or 3
 - 1
 - 2 or -3
 - none of these
10. The number of values of x which satisfy the equation $\frac{2x^2-10x}{x^2-5x} = x - 3$ is
- one
 - three
 - two
 - zero
11. (Tie Breaker) A careless teacher with 4 students has 4 report cards, one for each student. The teacher hand delivers the report cards to the students' lockers (here of course each student has his or her own locker). How many ways can the teacher place all the report cards in the incorrect lockers, with one and only one report card being placed in each locker?