Math 102: Sample Exam 1

1. How many odd whole numbers need to be added to get a sum of
   (a) 81 **answer:** 9
   (b) 169 **answer:** 13
   (c) 529 **answer:** 23
   (d) n^2 **answer:** n
2. Find the sum
   (a) $2+4+6+\cdots+900$ **answer:** 202,950
   (b) $1+2+3+\cdots+900$ **answer:** 405,450
   (c) $1+3+5+\cdots+899$ **answer:** 202,500
3. Consider the following differences

$$6^2 - 5^2 = 11$$

$$56^2 - 45^2 = 1111$$

$$556^2 - 445^2 = 111,111$$

   (a) Predict the next line in the sequence of differences. Use your calculator to check your answer. **answer:** 11,111,111
   (b) What do you think the 8th line will be? **answer:** 1,111,111,111,111,111
   (c) What do you think the n^{th} line will be? **answer:** An integer whose decimal representation has $2n$ 1's.
4. On a balance scale, two spools and one thimble balance 8 buttons. Also, one spool balances one thimble and one button. How many buttons will balance one spool? **answer:** 3
5. Mike said that when he opened his book, the product of the page numbers of the 2 facing pages was 7007. Without performing any calculations, prove that he was wrong. **Hint:** What is the product of two consecutive integers? Try some products. What do you notice?
6. Find the next three terms in the following sequences
   (a) $73, 82, 91, 100, \cdots$ **answer:** $109; 118; 127$
   (b) $1, 10, 100, \cdots$ **answer:** $1000; 10,000; 100,000$
   (c) $117, 229, 341, 453, \cdots$ **answer:** $565; 677; 789$
   (d) $1, 4, 9, 16, \cdots$ **answer:** $25, 36, 49$
   (e) $3, 7, 12, 18, 25, \cdots$ **answer:** $33, 42, 52$

7. Let $A = \{a, b, c\}, B = \{b, c\}, C = \{e\}$ and $U = \{a, b, c, d, e, f, g\}$. Draw a Venn diagram and use this to find $A \cap B$, $B \cup C$, $A \setminus B$ and A^c. **answers:** $\{b, c\}$, $\{b, c, e\}$, $\{a\}$, $\{d, e, f, g\}$.

8. True or False:

   (a) $7 \in \{6, 7, 8, 9\}$ **answer:** T

   (b) $5 \notin \{2, 3, 4, 6\}$ **answer:** T

   (c) $\phi \subseteq \{1\}$ **answer:** T

   (d) $\{1, 3\} \subset \{1, 3\}$ **answer:** F

   (e) $\{1, 3\} \subseteq \{1, 3\}$ **answer:** T

9. How many different one-to-one correspondences are possible between $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$? Also, find them. **answer:** $3 \cdot 2 \cdot 1 = 6$

10. Explain why there cannot be a one-to-one correspondence between $\{1, 2, 3, 4\}$ and $\{5, 6, 7\}$. **Hint:** Cardinality, and its definition.

11. What is the cardinality of

$$A = \{n \in \mathbb{N} : 7 < n \leq 28, \quad n \text{ is odd}\}?$$

    answer: 10

12. What are the cardinality of $A \cup B$ and $A \cap B$ where

$$A = \{1, 2, 3, 4\} \quad B = \{2, 4, 5, 6, a, z\}?$$

    answer: 8; 2 respectively

13. Is the following syllogistic argument valid? Please explain.

    All $A's$ are not $B's$.
    Some $B's$ are not $C's$.
    Therefore, Some $A's$ are $C's$.
    answer: No.

14. Is the following syllogistic argument valid? Please explain.

    Some $A's$ are not $B's$.
    All $A's$ are $C's$.
    Therefore, Some $C's$ are not $B's$.
    answer: Yes.

15. Find the contrapositive, converse and inverse of the following conditional proposition:

If I love mathematics, then I love Number Systems.

Also which are logically equivalent?

Answers:

- *Contrapositive:* If I don't love Number Systems, then I don't love mathematics.
- *Converse:* If I love Number Systems, then I love mathematics.
- *Inverse:* If I don't love mathematics, then I don't love Number Systems.

The conditional and the contrapositive of the conditional are logically equivalent. The inverse and converse of a conditional are logically equivalent. However, the conditional and its converse are not logically equivalent.