Math 107 Sample Exam Number 2, Spring 2010

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1 Random Variables

1. An insurance company sells a 100,000 dollar policy it promises to pay the policy holder in the event they are disabled and can no longer work. The probability that a policy holder will be paid for loss of job due to medical reasons is 1 in 500. Should the insurance company expect to earn a profit if it sells the policies for 250 dollars each?

   Answer: Let X be the random variable that measures how much the insurance company brings in on a policy. Consider the table:

x	$f(x)$	$x \cdot f(x)$
-99750	0.002	-199.5
250	0.998	249.5

   Summing the right column, we get

   $$E(X) = 50.$$

   Thus the insurance company will bring in, on average, 50 dollars per policy.

2. Let X be a random variable for an experiment. Suppose that X has the following probability density function:

x	$f(x)$
-1	0.9
0	0.05
10	0.04
100	0.01

   Find $E(X)$ and $sd(X)$.

Answers:

x	$f(x)$	$x \cdot f(x)$	$x - E(X)$	$(x - E(X))^2$	$(x - E(X))^2 \cdot f(x)$
-1	0.9	-0.9	-1.5	2.25	2.025
0	0.05	0	-0.5	0.25	0.0125
10	0.04	0.4	9.5	90.25	3.61
100	0.01	1	99.5	9900.25	99.0025
SUM		$E(X) = 0.5$			104.65

Thus $E(X) = 0.5$ and $sd(X) = \sqrt{104.65} \approx 10.2298583$.

3. Suppose the random variable X defined in the previous problem represents the winnings or losses upon playing a game of chance. Is this game fair? If you were asked if you wanted to play it, would you choose to do so often? Also, Find $P(win \quad money)$, $P(loose \quad money)$

   Answer: No, since $E(X) \neq 0$. Since $E(X) > 0$ you should play it as much as possible. $P(win \quad money) = 0.05$ and $P(loose \quad money) = 0.9$.

4. Suppose 25 percent of the items produced by a factory are defective. Suppose that 6 are chosen at random. Let X be the number of defective items in the collection of 6 that were chosen. Find:

   (a) $P(X = 2)$

   (b) $P(X < 2)$

   (c) $P(X > 2)$

   (d) $P(X \geq 2)$

   (e) $E(X)$

   (f) $sd(X)$

   Answers: 0.2966309, 0.5339355, 0.1694336, 0.4660644, 1.5, 1.0606066.

5. If for the above factory, 100 items are chosen. Again letting X be the number of defective items, use Chebyshev's inequality to find a lower bound for

$$P(10 < X < 40).$$

   Answer: Here $E(X) = 25, sd(X) = \sqrt{18.75} \approx 4.33$. Thus

$$P(10 < X < 40) \geq 1 - \frac{1}{(\frac{15}{4.33})^2} = 1 - \frac{1}{12}$$

2 Linear Regression

For the given data set:

$$(0,3),(1,3),(2,2),(3,2)$$

1. Draw a scatter plot.
2. Find an equation for the line of best fit, and graph it on the scatter plot.
3. Find the correlation coefficient.

3 The χ^2 test for Independence

Drinking and Crime: The sample data below were used by the English statistician Karl Pearson in 1909 to analyze the dependence of various crimes on the drinking habits of the criminal.

	Drinker	Abstainer	Total
Arson	50	43	
Rape	88	62	
Violence	155	110	
Stealing	379	300	
Coining	18	14	
Fraud	63	144	
Total			

At the $\alpha = 0.05$ significance level, test the hypothesis that alcohol use is independent of the type of crime committed.

For the solutions, see the corresponding excel file.