Math 107-04,07 Sample Final Exam, Spring 2010

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1 Descriptive Statistics

Given the data set

$$20, 50, 60, 70, 71, 72, 73, 73, 80, 89$$

Find:

1. The mean

   Answer: 65.8

2. The median

   Answer: 71.5

3. The standard deviation (by either technique) and the variance

   Answers: 19.17058, 367.5111

4. The $100 \cdot p$th percentile for

   (a) $p = 0.1$
   (b) $p = 0.17$
   (c) $p = 0.25$
   (d) $p = 0.5$
   (e) $p = 0.75$

   Answers: 35, 50, 60, 71.5, 73

5. Use a box and whisker argument to find any outliers.

   Answer: 20 is an outlier, since it does not lie in the interval [40.5, 92.5].

2 Probability

1. Suppose you toss a "fair" 20 sided die (in the shape of a icosahedron) with the faces labeled 1, 2, ... , 20. Find the probability that:

   (a) You roll a multiple of 3.

   (b) You don't roll a multiple of 3.

   Answers: $\frac{6}{20}$ and $\frac{14}{20}$ respectively.

2. For the sample space

$$S = \{a, b, c, d\},$$

   do the following tables define a probability distribution? Please explain.

(a)

outcome	probability
a	0
b	0
c	0
d	1

(b)

outcome	probability
a	-0.001
b	0
c	0.001
d	1

(c)

outcome	probability
a	0.23
b	0.23
c	0.23
d	0.29

(d)

outcome	probability
a	0.3
b	0.3
c	0.2
d	0.22

(e)

outcome	probability
a	0.25
b	0.5
c	0.15
d	0.1

   Answers: Yes, No, No, No and Yes respectively.

3. Suppose an experiment has two events A, B with the following probabilities

$$P(A) = \frac{1}{3}, P(B) = \frac{1}{2}, P(A \cap B^c) = \frac{1}{6}$$

Find:

(a) $P(A \cap B)$
(b) $P(A^c)$
(c) $P(B^c)$
(d) $P(A \cup B)$
(e) $P(A^c \cup B^c)$

Answers: $\frac{1}{6}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{6}$ respectively. To see these, use a Venn diagram as a visual aid.

4. Suppose A and B are independent events with probabilities:

$$P(A) = 0.3, P(B) = 0.6$$

Find:

(a) $P(A \cap B)$
(b) $P(A \cup B)$
(c) $P(A^c \cup B^c)$
(d) $P((A \cup B)^c)$
(e) $P(A|B)$

Answers: 0.18, 0.72, 0.82, 0.28 and 0.3 respectively. To see these, use a Venn diagram as a visual aid.

5. Suppose A and B are events with

$$P(A) = 0.35, P(B) = 0.25, P(A \cap B) = 0.2$$

(a) Find $P(A|B)$.
(b) Find $P(B|A)$.
(c) Are A, B independent events? Please explain.

Answers: 0.8, 0.5714, and No respectively.

6. In a math course suppose that 25 percent of the students failed the first exam, 15 percent failed the second exam and 5 percent failed both exams.

(a) Are the events failing the first exam and failing the second exam independent?
(b) Find P(Failed second exam given failed first exam).

Answers: No and 0.2 respectively.

3 Linear Regression

Given the data set

$$(1,75),(2,100),(3,65),(4,72),(6,45)$$

Find:

1. The line of best-fit.
2. Pearson's correlation coefficient.

Answers: $S_{xx} = 14.8$ $S_{yy} = 1569.2$, $S_{xy} = -114.4$, $\bar{x} = 3.2$, $\bar{y} = 71.4$, $m = -7.72973$, $b = 96.13514$, $r = -0.75068$.

Thus $y = -7.72973x + 96.73514$.

4 Multivariate Categorical Data and Independence

A random group of 300 adult males was cross-classified according to age and cholesterol level. The following table has the results:

Age vrs Cholesterol level	Low	Medium	high cholesterol	total
20-34 yrs.	66	24	8	98
35-54 yrs.	54	48	22	124
55-74 yrs.	18	50	10	78
total	138	122	40	300

Using the χ^2 test for independence, at the $\alpha = 0.05$ level, test the hypothesis that age and cholesterol level are independent. **Answer:** $\chi^2 = 39.526$, and thus we reject H_o in favor of H_1 : dependence.

5 Random Variables

1. Let X be a random variable with probability density function $f(x)$ given by

x	$f(x)$
0	0.1
1	0.1
2	0.4
3	0.15
4	0.15
5	0.1

Find:

(a) $P(X \text{ is even })$

(b) $E(X)$

(c) $sd(X)$

Answers: 0.65, 2.45, 1.40

2. Suppose 10 percent of the items produced by a factory are defective. Suppose that 8 are chosen at random. Let X be the number of defective items in the collection of 8 that were chosen. Find:

   (a) $P(X = 4)$
   (b) $P(X < 2)$
   (c) $P(X \geq 2)$
   (d) $P(X = 0)$
   (e) $P(X > 0)$
   (f) $E(X)$
   (g) $sd(X)$

   Answers: 0.0046, 0.8131, 0.1869, 0.4305, 0.5695, 0.8, 0.8485

3. If for the above factory, 100 items are chosen. Again letting X be the number of defective items, use Chebyshev's inequality to find a lower bound for

$$P(0 < X < 20).$$

   Answer: 0.91

6 The Normal Distribution

1. Suppose the heights of American males are (approximately) normally distributed with a mean $\mu = 68$ inches and standard deviation $\sigma = 2.5$ inches. Let $X(male) = height$ in inches. Find:

   (a) $P(60 < X < 65)$
   (b) $P(63 < X \leq 73)$
   (c) $P(X \geq 72)$
   (d) $P(X > 72)$
   (e) $P(X \leq 65)$

   Answers: 0.114, 0.954, 0.055, 0.055, 0.115

2. Suppose that a random variable X has expected value of $E(X) = 35$ and standard deviation $sd(X) = 5$. Suppose that a sample of 64 elements was chosen and

$$\bar{X} = \frac{X_1 + X_2 + \cdots + X_{63} + X_{64}}{64}$$

is the sample average. Suppose that one observes $\bar{X} = 33.2$. Assuming $sd(X)$ is correct, use the central limit theorem to test the claim that $E(X) = 35$ at the $\alpha = 0.05$ significance level.

Answer: Consider $Z = \frac{\bar{X}-E(X)}{\frac{sd(X)}{\sqrt{n}}} = \frac{\bar{X}-35}{\frac{5}{8}}$. Here we need Z to have its values in the region of 95% probability symmetric about $Z = 0$. This is $[-1.96, 1.96]$ Thus using $\bar{X} = 33.2$ we see $Z = \frac{33.2-35}{\frac{5}{8}} = -2.88$. This is not in the desired interval, and so we reject the hypothesis that $E(X) = 35$.