Math 112: Rational Exponents and Complex Numbers

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n-th Roots:

Given a non-negative number x, we can define the **square root** of x to be the number non-negative number y so that $y^2 = x$. We usually use

$$\sqrt{x}$$

to denote the square root of x.

Examples:

- $$\sqrt{25} = 5$$

- $$\sqrt{100} = 10$$

- $$\sqrt{\frac{1}{16}} = \frac{1}{4}$$

- $$\sqrt{2} = 1.41421356237\cdots$$

We may define the **cube root** of any real number x to be the number y so that $y^3 = x$. We usually use

$$\sqrt[3]{x}$$

to denote the cube root of x. Note that x and $\sqrt[3]{x}$ have the same sign.

Examples:

- $$\sqrt[3]{125} = 5$$

- $$\sqrt[3]{-125} = -5$$

- $$\sqrt[3]{1000} = 10$$

- $$\sqrt[3]{-1000} = -10$$

- $$\sqrt[3]{\frac{1}{64}} = \frac{1}{4}$$

- $$\sqrt[3]{\frac{-1}{64}} = \frac{-1}{4}$$

- $$\sqrt[3]{2} = 1.25992104989\cdots$$

Likewise, for any positive integer, we can define the so-called n-**th root**.

If n is even and positive, given a non-negative number x, we define the n-th root of x to be the non-negative number y so that $y^n = x$.

If n is odd and positive, given a number x, we define the n-th root of x to be the number y so that $y^n = x$. We usually use

$$\sqrt[n]{x}$$

to denote the n-th root of x.

Note that in the case where n is odd, the n-th root $\sqrt[n]{x}$ has the same sign as x.

We now note an important property of n-th roots, which is defined for all real numbers x:

- For n a positive odd number

$$\sqrt[n]{x^n} = x$$

- For n a positive even number

$$\sqrt[n]{x^n} = |x|$$

In lecture we shall examine the functions

$$f(x) = \sqrt{x}, \quad g(x) = \sqrt[3]{x}$$

by drawing rough sketches of their graphs via plotting points.

In lecture we shall do some of the following exercises: page 419 numbers 4, 6, 8, 12-24 even, 28, 44-52 even, 78, 80

Homework: page 419 numbers 3-27 odd, 43-51 odd, 77, 79

Rational Exponents:

Given any positive integer n we define

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

whenever $\sqrt[n]{x}$ is defined.

Given any two positive integers m, n whose greatest common divisor is 1, we define

$$x^{\frac{m}{n}} = (\sqrt[n]{x})^m$$

provided $\sqrt[n]{x}$ exists. We note that in such a case, the following holds:

$$x^{\frac{m}{n}} = (\sqrt[n]{x})^m = \sqrt[n]{x^m}.$$

Given any two positive integers m, n whose greatest common divisor is 1, we define

$$x^{-\frac{m}{n}} = \frac{1}{x^{\frac{m}{n}}}$$

provided $\sqrt[n]{x}$ exists and is not 0.

Exponents:

The following are some properties of exponents. We assume that m, n are rational numbers, and a, b are real numbers for which all expressions in the below equalities exist.

- $$a^n a^m = a^{n+m}.$$

- $$\frac{a^n}{a^m} = a^{n-m}$$

- $$a^0 = 1$$

- $$(a^m)^n = a^{m \cdot n}$$

- $$(a \cdot b)^n = a^n \cdot b^n$$

- $$(\frac{a}{b})^n = \frac{a^n}{b^n}$$

- $$a^{-n} = \frac{1}{a^n}$$

In lecture we shall do some of the following exercises: pages 426-427 numbers 2, 4, 6, 8, 14, 16, 20, 22, 24, 32, 34, 38, 44, 48, 50, 54, 58, 60; page 435 numbers 2, 4, 6, 12, 16, 18, 32, 34, 36, 40, 42, 44

Homework: pages 426-427 numbers 1-59 odd; page 435 numbers 1-7 odd, 13, 15, 17, 31-49 odd

Solving Equations Involving Radical Expressions:

In lecture we shall do some of the following exercises: pages 456-457 numbers 2-22 even, 52-62 even

Homework: pages 456-457 numbers 1-21 odd, 51-63 odd