Math 113: Trigonometry

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The Distance Formula and Equations of Circles:

Given two points (a, b) and (x, y) in the plane, we define the Euclidean distance or simply **distance** between them to be

$$d = \sqrt{(x-a)^2 + (y-b)^2}.$$

A **circle** with **center** (a, b) and **radius** r is simply the collection of all points (x, y) do that the distance between (a, b) and (x, y) is r. Thus (x, y) must satisfy:

$$r = \sqrt{(x-a)^2 + (y-b)^2}$$

or simply

$$(x-a)^2 + (y-b)^2 = r^2.$$

The Unit Circle:

We now consider the the set of all points (x, y) in the plane that satisfy the equation

$$x^2 + y^2 = 1.$$

The **unit circle** is the collection of the points in the plane that lie on the circle centered at the origin $(0, 0)$ with radius 1.

Measuring Angles in Geometry:

The are two units we commonly use to measure angles in geometry. There is the unit called a **degree**, denoted with $^\circ$, and there is the unit called a **radian**, denoted with rad.

The conversion between the two is

$$360^\circ = 2\pi \quad rad.$$

Here

$$\pi = 3.14159265358979323846 26 \cdots$$

is the irrational number that measures the ratio between the circumference of a circle, and its diameter.

The radian measure of an angle θ actually is very natural. It measure the arclength of the of the portion of the unit circle whose angle measure is θ.

The Trigonometric Functions:

Given an angle θ, we define the functions **cosine** and **sine** of θ as the x and y coordinates of the point on the unit circle which you obtain by rotating from the positive x-axis by θ in the counterclockwise direction. Thus

$$(\cos\theta, \sin\theta)$$

represents this point. Where

$$f(\theta) = \cos\theta$$

is the **cosine** function, and

$$g(\theta) = \sin\theta$$

is the **sine** function.

Hence, $\sin\theta$ and $\cos\theta$ must satisfy the so-called **Pythagorean identity**

$$(\sin\theta)^2 + (\cos\theta)^2 = 1,$$

which is sometimes written as

$$\sin^2\theta + \cos^2\theta = 1.$$

By the definition of the trigonometric functions sine and cosine, we see that sine and cosine are periodic with period 2π radians or 360°. That is

$$\cos\theta = \cos(\theta + 2\pi), \quad \sin\theta = \sin(\theta + 2\pi)$$

for any angle θ in radians or

$$\cos\theta = \cos(\theta + 360^{\circ}), \quad \sin\theta = \sin(\theta + 360^{\circ})$$

for θ in $degrees$.

We define the trigonometric functions **tangent, cotangent, secant, cosecant** of θ respectively, by the identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}, \quad \csc\theta = \frac{1}{\sin\theta}.$$

Some values of θ, for which **you will be required to know the values of the trigonometric functions** are

$$\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}.$$

In lecture, I shall describe how to obtain the values of the trigonometric functions for these angles.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

Some Important Identities and properties:

1.

$$\sin\theta = \sin(\theta + 2\pi), \quad \cos\theta = \cos(\theta + 2\pi).$$

2. $\sin\theta$ is odd and $\cos\theta$ is even.

3.

$$\cos(\theta + \pi) = -\cos\theta, \quad \sin(\theta + \pi) = -\sin\theta.$$

4.

$$\sin(\frac{\pi}{2} - \theta) = \cos\theta, \quad \cos(\frac{\pi}{2} - \theta) = \sin\theta.$$