Analysis of the gravity shield engine described at
“http://www.lhup.edu/
/~dsimanek/museum/unwork.htm#gravshld”

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This is my analysis of the proposed perpetual motion device entitled “The Gravity shield engine” on your website “http://www.lhup.edu/~dsimanek/museum/unwork.htm#gravshld”. I think it is a very interesting problem, however, I found the discussion on “http://www.lhup.edu/~dsimanek/museum/gravshld.htm” to be wanting, and so I went about trying to solve the problem myself. My solution is in the first section. I think it is satisfying and pedagogical, in that it is a simple, direct calculation. In the end, the conclusion is that the assumed gravitational field produced by a “gravitational shield” is in fact incomplete. I first show that the assumed field does in fact generate perpetual motion, but when the field is derived from a scalar potential, additional horizontal components of the gravitational field are generated at the border between the shielded and non shielded region. This horizontal gravitational field applies a torque which exactly cancels out the torque created by the vertical gravitational field in the non shielded region. I also believe that the arguments presented “http://www.lhup.edu/~dsimanek/museum/gravshld.htm” suffer from a conceptual error which I point out in the final section.

Solution via direct calculation of torque on wheel

A wheel has a frictionless axle. Now just insert a gravity shield under one side, making that side lighter and this will initiate and maintain rotation. Indeed, you’d better extract energy from it continually, or put a brake on it, or it will spin so fast it will tear itself apart.

The supposed gravitational field for the engine can be written as the follows:

$$\mathbf{F}(x, y) = -\Theta(x)g\hat{y}$$  \hspace{1cm} (1)

where $\hat{y}$ is the unit vector pointing in the positive $y$ direction, $g$ is the gravitational constant at the surface of the earth, and $\Theta(x)$ is Heaviside’s step function defined by:

$$\Theta(x) = \begin{cases} 
1 & : x \geq 0 \\
0 & : x < 0 
\end{cases} \hspace{1cm} (2)$$

One can then calculate the torque on the wheel$^1$:

$$\tau = g \frac{m}{\pi R^2} \int_0^R dr \int_0^\pi d\theta \sin\theta = g \frac{2m}{3\pi} R \hspace{1cm} (3)$$

where $m$ is the mass of the wheel and $R$ is the wheel’s radius. The above calculation shows that this force field does in fact exert a positive torque on the object, and thus does seem to lead to perpetual motion.

What is going on here? The fact that this system can perform an infinite amount of work leads one to expect that there is no energy function for this system. In fact this can easily be confirmed by calculating curl of (1):

$$d\tau = r \cdot |\mathbf{F}| \cdot \sin\theta = r \cdot g \left( \frac{m}{\pi R^2} \right) dx dy \cdot \sin\theta = r \cdot g \left( \frac{m}{\pi R^2} \right) r dr d\theta \cdot \sin\theta$$

\hspace{1cm}$^1$
\[ \nabla \times \mathbf{F}(x, y) = \left( \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) \mathbf{\hat{x}} + \left( \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) \mathbf{\hat{y}} + \left( \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \mathbf{\hat{z}} \]
\[ = -g \frac{d\Theta(x)}{dx} \mathbf{\hat{z}} = -g\delta(x)\mathbf{\hat{z}} \neq 0 \quad (4) \]

where \( \delta(x) \) is the Dirac delta function defined by:
\[ \int_a^b \delta(x)f(x)dx = \begin{cases} f(0) & : a \leq 0 \leq b \\ 0 & : a < 0, b < 0 \\ 0 & : a > 0, b > 0 \end{cases} \quad (5) \]

where \( f \) is an arbitrary continuous function. Since the field is not curl-less, it can’t be derived from a scalar potential.

This is all well and good, but suppose we did in fact have a “gravitational shield”. What type of gravitational field could it be used to produce? To have a well defined potential energy, as required by Newton’s theory of gravity, we need to be able to derive the gravitational field from a scalar potential function \( \Phi(x, y) \). Let us try to write a potential that gives force field (1) in the regions of \( x > 0 \) and \( x < 0 \). The obvious choice is
\[ \Phi(x, y) = \Theta(x)gy. \]

Calculating the force we now have
\[ \mathbf{F}(x, y) = -\nabla \Phi(x, y) = -\frac{d\Theta(x)}{dx}gy\mathbf{\hat{x}} - \Theta(x)g\mathbf{\hat{y}} \]
\[ = -\delta(x)gy \mathbf{\hat{x}} - \Theta(x)g\mathbf{\hat{y}}. \quad (8) \]

Comparing (1) and (8), we see that the two fields are the same for \( x \neq 0 \), but that we have an additional field component in the \( x \) direction at \( x = 0 \) in (8). This added field gives an additional contribution to the torque. The total torque on the disk is the sum of the torque computed using (1), \( \tau \), and the torque resulting from this additional component of the field given in (8), \( \tau' \):
\[ \tau_{\text{total}} = \tau + \tau'. \quad (9) \]

Calculating \( \tau' \) we find:
\[ \tau' = -g \frac{m}{\pi R^2} \int_{-\epsilon}^{\epsilon} dx \delta(x) \int_{-R}^{R} dy y^2 = -g \frac{m}{\pi R^2} \times \frac{2}{3} R^3 = -\tau. \quad (10) \]

Thus the total torque on the wheel is zero hence the wheel will not accelerate.
Errors in analysis of section “Just what would a gravitational shield do?”

I think the section “Just what would a gravitational shield do?” suffers from a conceptual error which invalidates the section’s argument. Specifically, I believe that the sentences

The mass moves down in the earth’s field, and up an equal distance in the earth’s field, so the work done on it by the earth and the work done by it on the earth are equal in size.

and

As the mass moves down, the source of the gravitational field (the earth) does work on it. As the mass moves up, the mass does an equal amount of work on the source of the gravitational field.

are incorrect. Work is force times distance. Since the earth effectively doesn’t move, and the force exerted on the earth by the mass is equal in magnitude to the force the earth exerts on the mass (Newton’s third law), and is thus finite, no work is done on the earth by the mass at any point in its rotation. By the same reasoning, the mass does no work on the gravity shield, which is stationary. I think that the conceptual error that is being made here is a misinterpretation of Newton’s third law. Newton’s third law states that forces come in pairs which are equal and opposite, not that work comes pairs which are equal and opposite.

As stated above the reason field (1) admits perpetual motion is that it is not conservative. The actual force consistent with Newton’s laws of gravitation is (8). The correct conservative field (8) has additional lateral components which give rise to a torque which cancel the torque caused by the vertical field in the $x > 0$ region. I believe this is the correct reason why the engine wouldn’t work as a perpetual motion machine (unless Newton’s law of gravitation is wrong).